$\mathbb{Q}$ = the set of rational numbers

$:= \left\{ \frac{m}{n} : m \in \mathbb{Z} \text{ and } n \in \mathbb{N} \right\}$

= "The set of all numbers of the form $\frac{m}{n}$, where $m \in \mathbb{Z}$ and $n \in \mathbb{N}$."

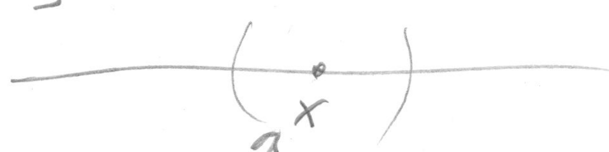
<u>Thm</u> S dense in $\mathbb{R}$ $\Longleftrightarrow$ every nonempty open interval contains a point of S.

<u>Ideas</u>

$\Longrightarrow$: Assume S dense. Given

$\exists s_k \to y$.

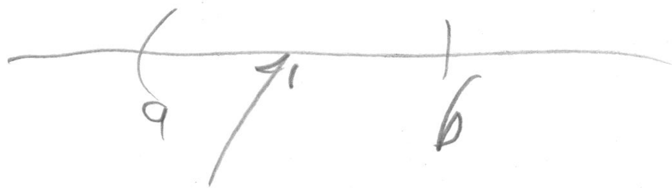
$\Longleftarrow$: Assume every open interval contains a point of S. Let $x \in \mathbb{R}$. Must find $\{s_n\}$ in S, $s_n \to x$.



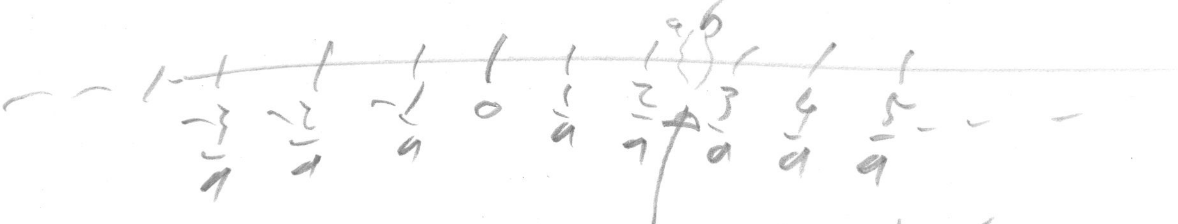
Pick a sequence of shrinking intervals, say $(x - \frac{1}{n}, x + \frac{1}{n})$

<u>Thm</u> $\mathbb{Q}$ is dense in $\mathbb{R}$.

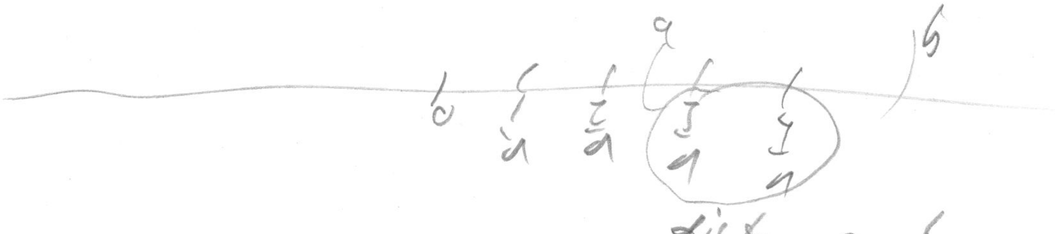
<u>Idea</u> Show any open interval (a,b) contains a rational.



Find m and n s.t. $a < \frac{m}{n} < b$.



No good; n is too small.



distance $\frac{1}{n}$.

So we want $\frac{1}{n} < b-a$.

Once you have n, trick is to cross multiply!

$$nb - na > 1$$

So $\exists\, m \in \mathbb{Z}$ s.t. $na < m < nb$.