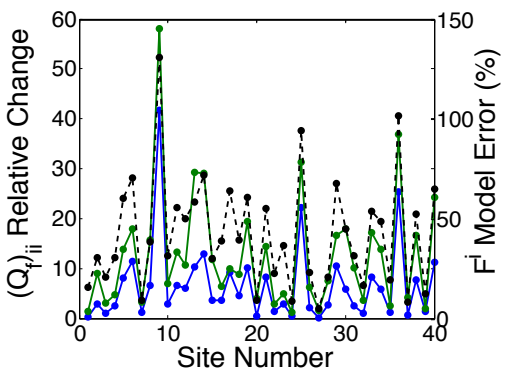
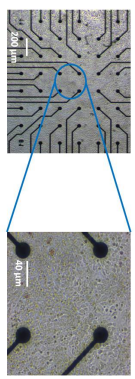
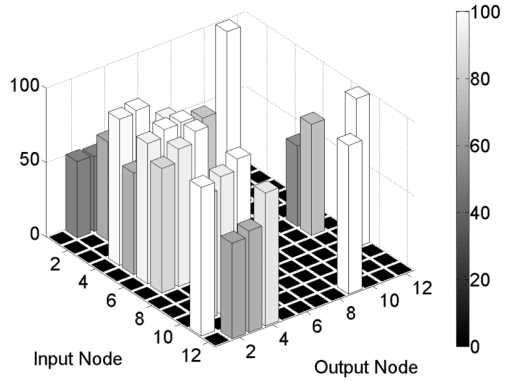
Data Assimilation

- Estimate state/parameters from noisy observations
- Requires noise statistics (typically unknown)
- Adaptive data assimilation:
  - Estimates noise statistics 'online'
  - Compensates for model error



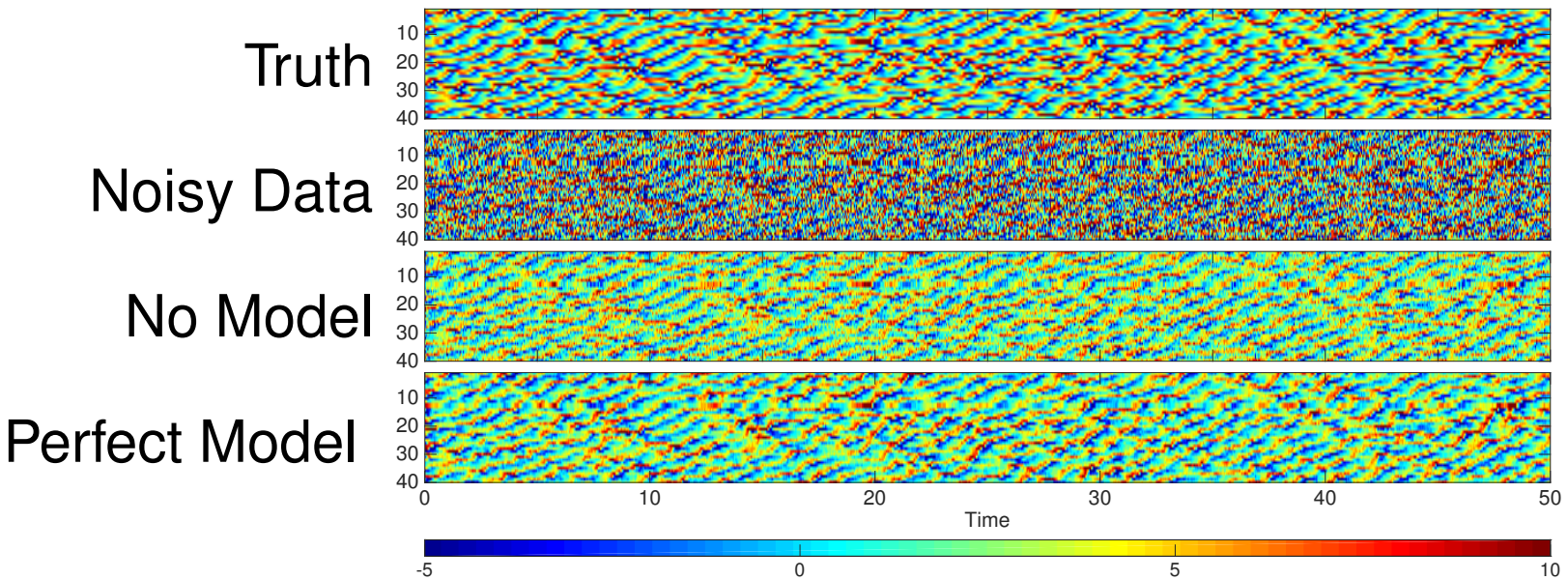




B. and Sauer (Tellus A, 2013)

KALMAN-TAKENS FILTER

Filtering without a model or partially known model



Diffusion Forecast

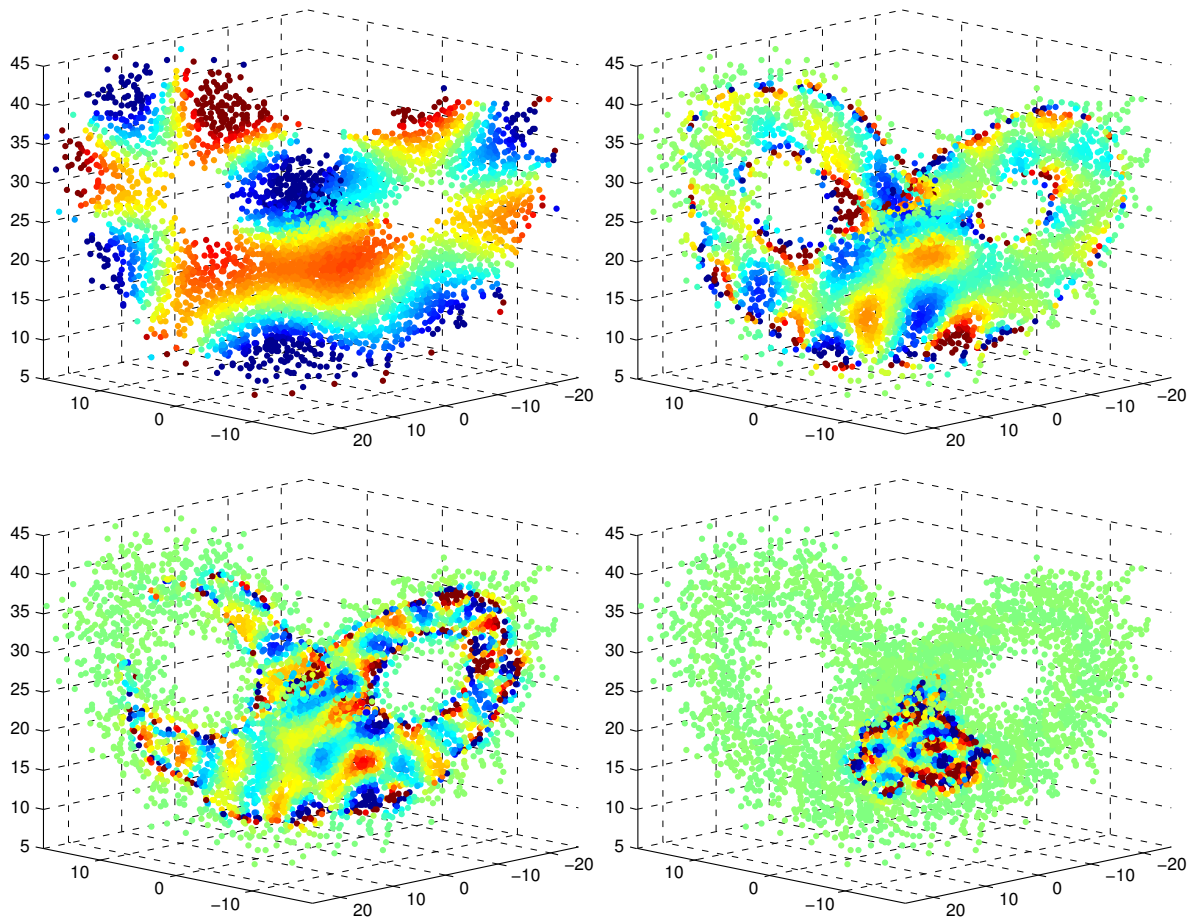
- Forecast is entirely data driven (model free)
- Predicts a probability distribution
- Estimates probabilities of extreme events
- Also used to correct model error

$$
\begin{array}{ccc}
p(x,t) & \xrightarrow{\text{Diffusion Forecast}} & p(x,t+\tau) = e^{\tau \mathcal{L}^*} p(x,t) \\
\Big\downarrow \langle p, \varphi_j \rangle & & \Big\uparrow \sum_j c_j \varphi_j q \\
\vec{c}(t) & \xrightarrow{A_{lj} \equiv \mathbb{E}[\langle \varphi_j, S\varphi_l \rangle_q]} & \vec{c}(t+\tau) = A\vec{c}(t).
\end{array}
$$

B., Harlim, and Giannakis (Physical Review E, 2015)

MANIFOLD LEARNING ⇒ CUSTOM 'FOURIER' BASIS

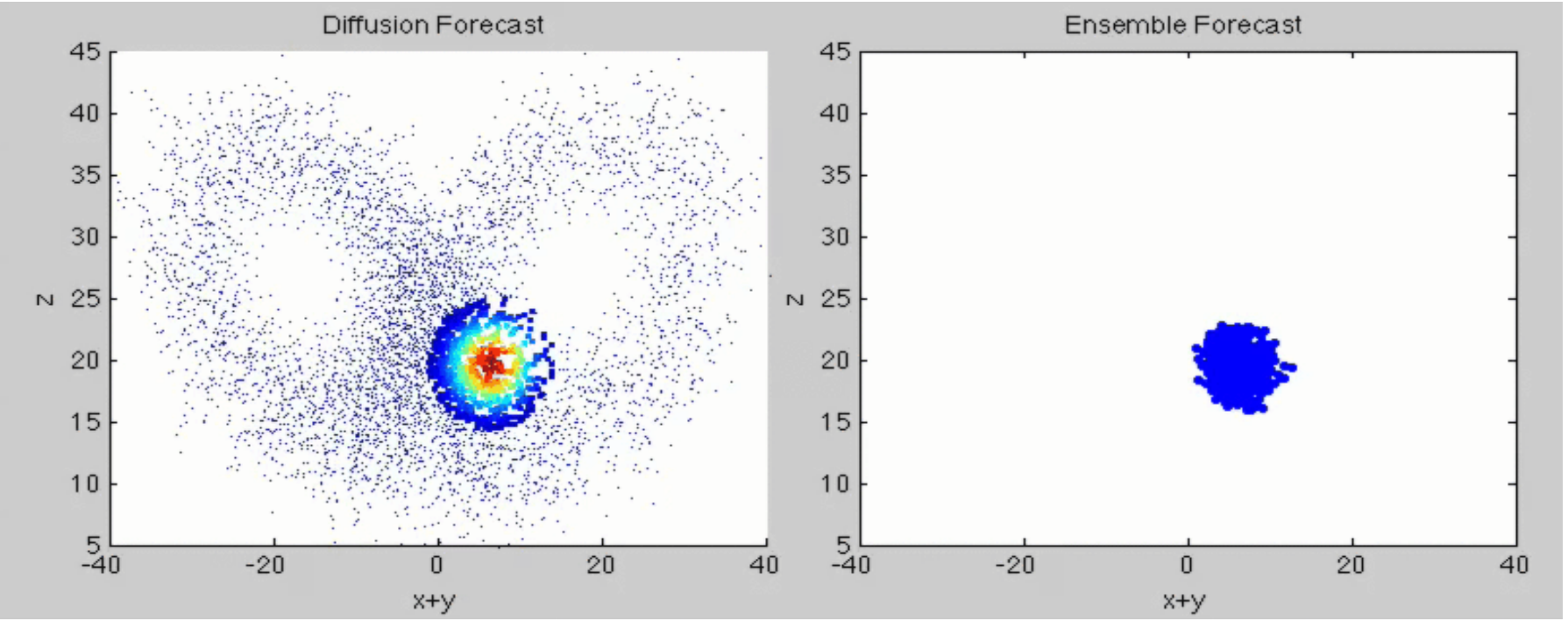
- **Optimal basis:** Minimum variance $A_{lj} \equiv \mathbb{E}[\langle \varphi_j, S\varphi_l \rangle_q]$



Diffusion Forecast Example

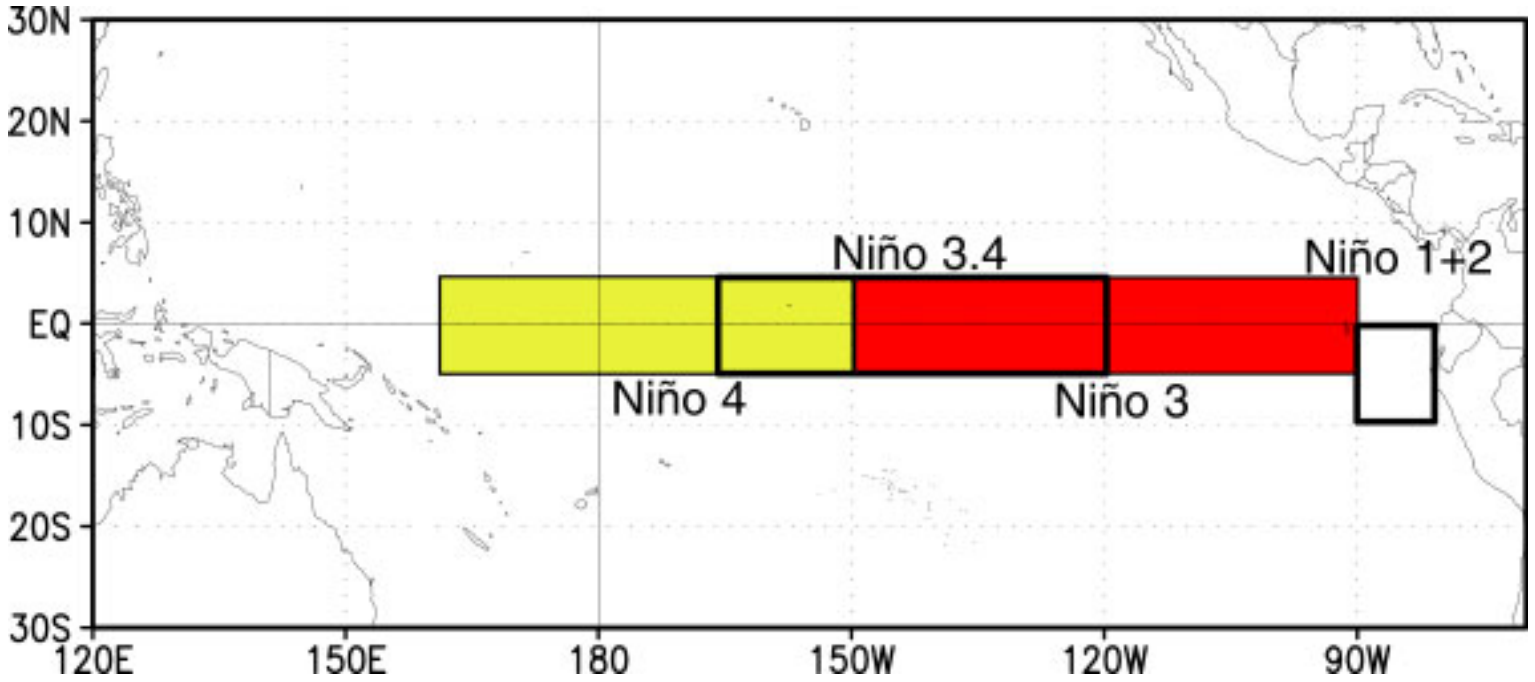
No Model

Perfect Model



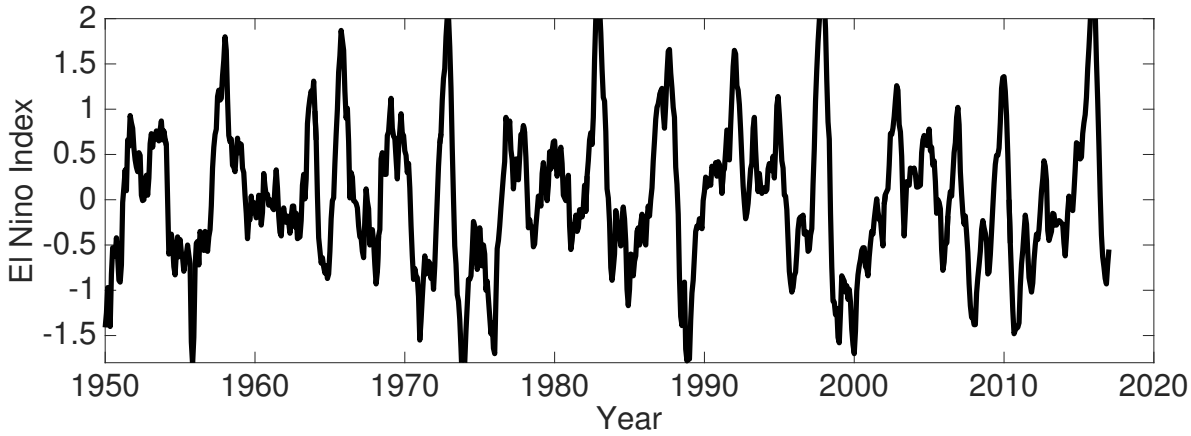
Forecasting the El Niño Index

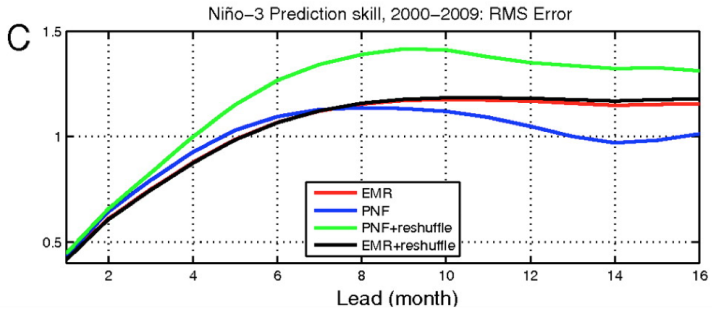
Sea surface temperatures (SST) in the Niño indices:



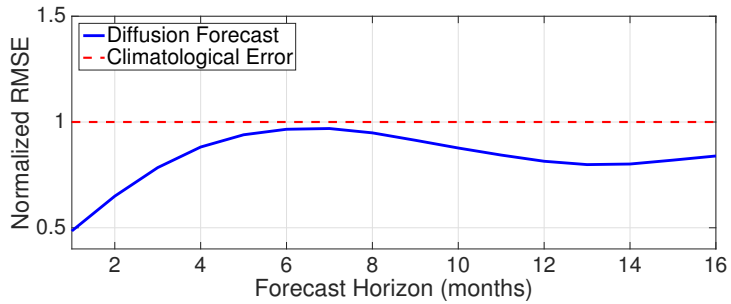
Index: 3-month running average SST anomaly

Forecasting the El Niño Index



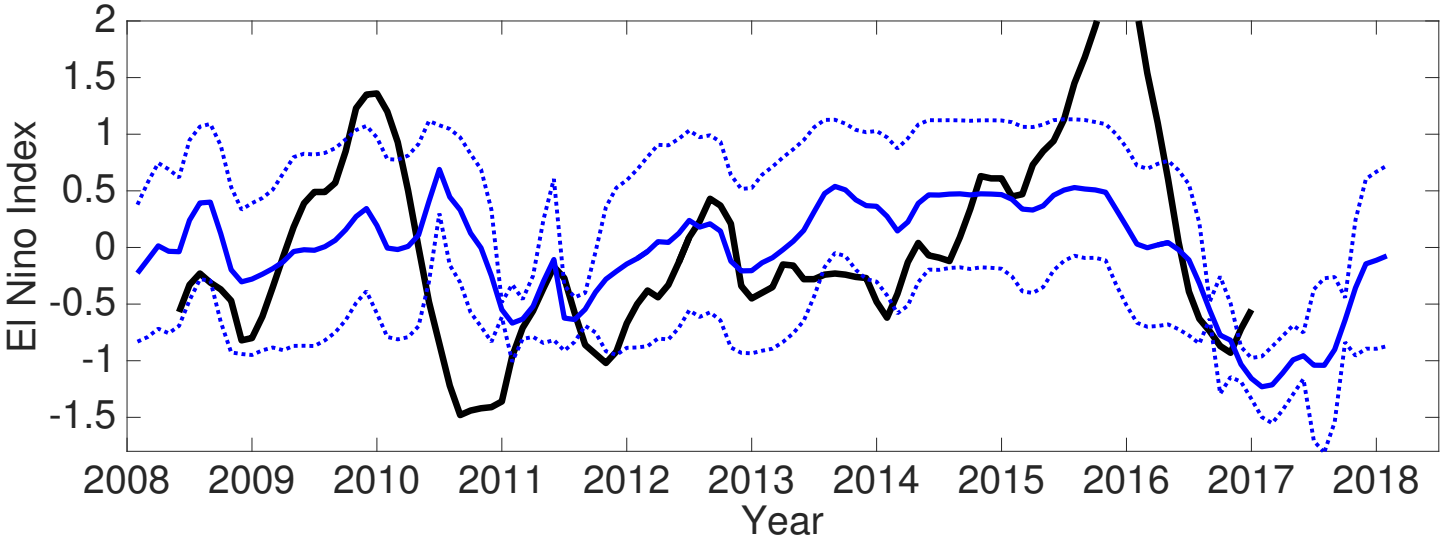


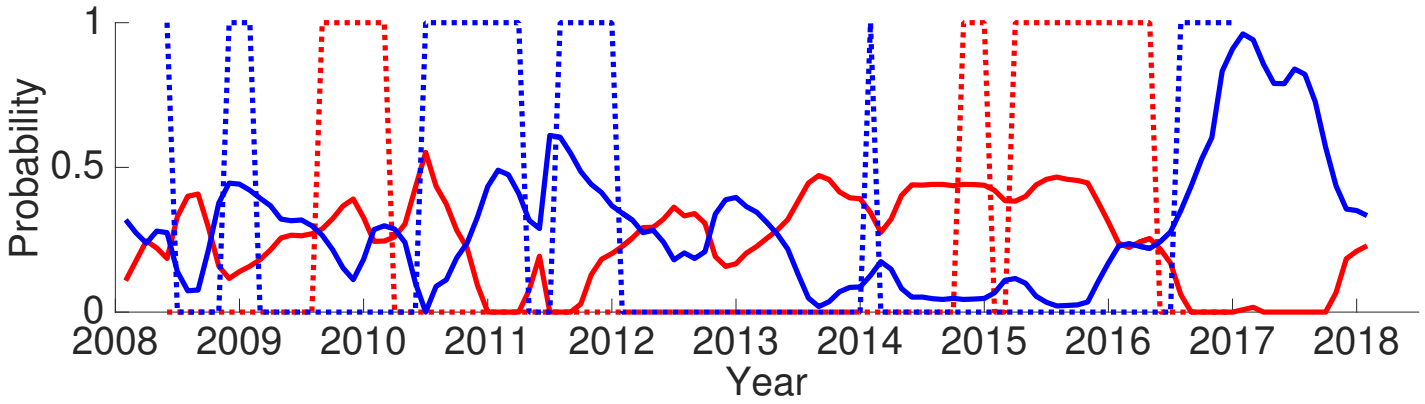
Chekrouna, Kondrashov, and Ghil, PNAS 2011,108,no.29



Diffusion Forecast

13-MONTH FORECAST





Probability of El Nino — Actual El Nino — Probability of La Nina — Actual La Nina

El Niño = Index > 0.5 La Niña = Index < -0.5