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ADDENDUM

Local fluid and heat flow near contact lines

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It has recently come to our attention that our paper, which describes Marangoni-driven flow near a contact line, overlooks solutions involving a general thermal boundary condition on the free surface (private communication, S. J. Tavener 1997). These new solutions are applicable for non-isothermal flows in a corner region where one boundary is a rigid plane (and either perfectly insulating or perfectly conducting) and the other is a free surface upon which a general thermal boundary condition is applied. We describe these additional solutions below.

Consider non-isothermal flow in a single wedge bounded by a rigid plane at $\theta = 0$ and a planar free surface at $\theta = \alpha$. We consider the cases where the boundary at $\theta = 0$ is either perfectly insulating (no flux) or perfectly conducting. On the free surface $\theta = \alpha$ we impose a general thermal boundary condition

$$\frac{k}{r}\frac{\partial T}{\partial \theta} = h(T - T_\infty), \tag{1}$$

where k is the thermal conductivity and h is the heat transfer coefficient. The local thermal field (before applying boundary conditions) has the general form given by equation (2.5).

When $\tau < 1$, where $T \sim r^\tau f(\theta)$, the free surface condition (1) leads to $\partial T/\partial\theta = 0$ (i.e. a no-flux boundary condition) to leading order. When $\tau > 1$ the free surface condition (1) again leads to the no-flux boundary condition with the additional condition that the temperature at $r = 0$ is T_∞. Solutions for these cases are described in our paper.

There are additional solutions when $\tau = 1$. Here, the general thermal boundary condition (1) does not reduce to the no-flux condition. When $\theta = 0$ is a no-flux boundary, the thermal field is given by

$$T = T_0 + \frac{(h/k)(T_\infty - T_0)}{\sin\alpha} r\cos\theta + O(r^2), \tag{2}$$

where T_0 is the temperature at the corner. If $\alpha = \pi$, the only solution of the form sought with $\tau = 1$ is $T = T_\infty$ (i.e. an isothermal corner). When $\theta = 0$ is a conducting boundary (at constant temperature T_0), the thermal field is given by

$$T = T_0 + \frac{(h/k)(T_0 - T_\infty)}{\cos\alpha} r\sin\theta + O(r^2). \tag{3}$$

When $\alpha = \pi/2$ or $3\pi/2$ equation (3) is replaced by

$$T = T_\infty + B_1\left(r\sin\theta + \frac{h}{2k}\frac{\sin\alpha}{\cos 2\alpha} r^2 \sin 2\theta\right) + O(r^3), \tag{4}$$

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where B_1 is arbitrary and T_0 must equal T_∞. Equations (2) and (3) correspond to isotherms perpendicular and parallel to the boundary $\theta = 0$, respectively, to leading order in r. The special case represented by equation (4) also corresponds to leading-order isotherms parallel to $\theta = 0$. The temperature gradient along the free surface $\partial T/\partial r|_\alpha$ is constant (to leading order in r) in each case.

The Marangoni flow (streamfunction form, 'partial local solution') driven by these thermal gradients satisfies $\nabla^4 \tilde{\psi}_p = 0$ and boundary conditions (2.19a, b, c). Since the temperature gradient (and by assumption the surface tension gradient) is constant to leading order in r, the corresponding streamfunction $\tilde{\psi}_p$ is proportional to r^2 and is given by equation (2.24). Here $f_1(\alpha)$ is interpreted to be $\partial T/\partial r|_\alpha$ as evaluated from equations (2) or (3). This flow corresponds to a locally-driven Marangoni flow only. The complete local flow is obtained by adding to this flow the additional local flow driven from far-field effects as described previously. There are no additional 'local solutions' that have a corner-driven Marangoni flow.

An additional correction to this previous paper is that in the first paragraph of §2.2.1, the equation $\nabla^2 \psi = 0$ should be replaced by the biharmonic equation $\nabla^4 \psi = 0$.

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